Implementing Quasisymmetric Surface Optimization in PyPlasmaOpt

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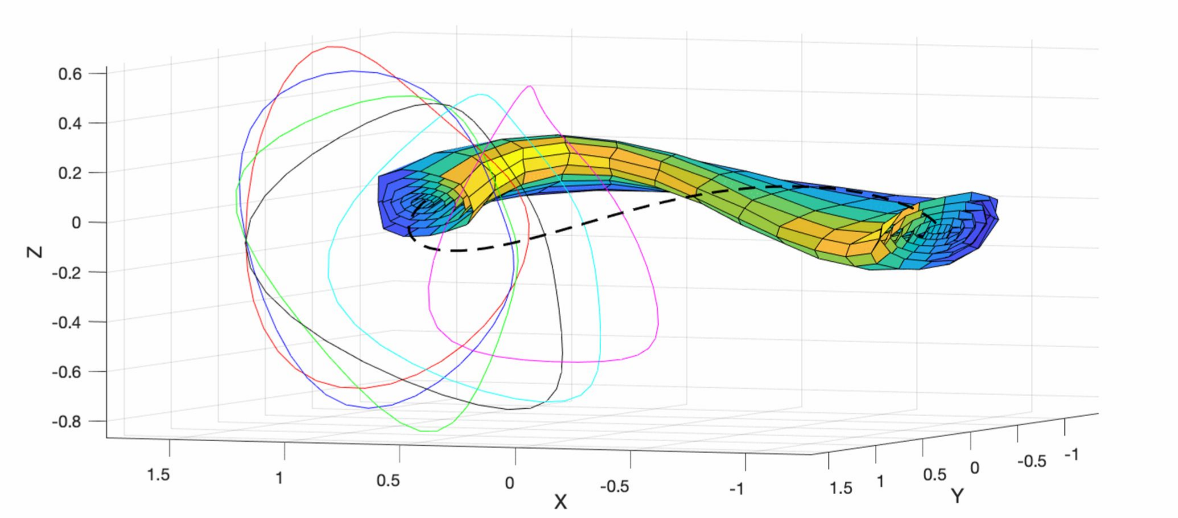


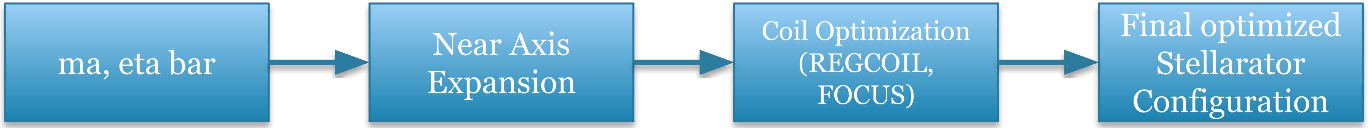
Figure courtesy A. Giuliani

PyPlasmaOpt: Overview

- Code for optimizing Stellarator vacuum field and coils
- Developed as part of Simons Foundation collaboration
- C++ and Python, connected with PyBind11

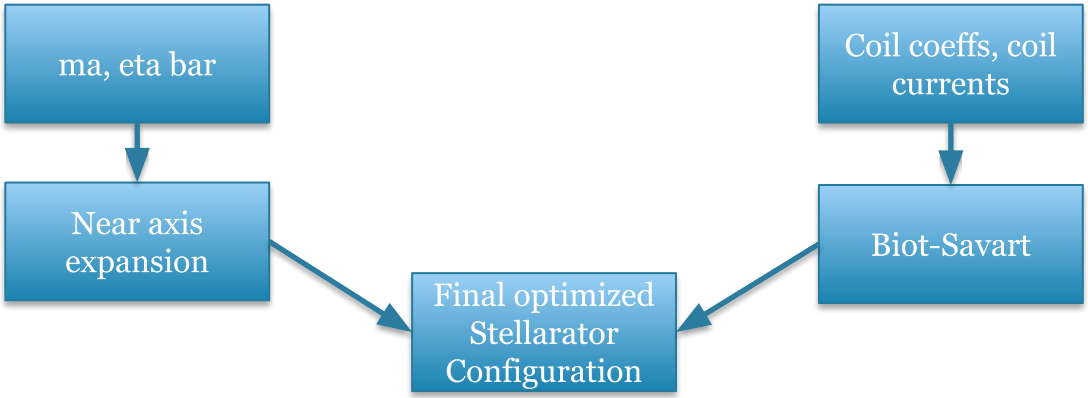
“Traditional” Approach to Stellarator Optimization

- Two distinct steps in optimization:
  - Optimization for physics properties (e.g. quasisymmetry, confinement)
  - Optimization for engineering properties (e.g. coil complexity, intercoil distance)
- Drawbacks – decoupling of physics and engineering could result in weaker overall optimization result.
- Maybe a different QS field has equivalent physics but superior engineering?



PyPlasmaOpt Approach to Stellarator Optimization

- Recent innovations in calculating QS fields analytically based on a near axis expansion (Landreman et al. 2019) allow for rapid calculations of desirable vacuum fields
  - DOFs: Magnetic axis, eta bar
- Field generated by initial simple coils using Biot-Savart
  - DOFs: Coil Fourier coefficients, coil currents, magnetic axis
- Generate an objective function that forces two fields to match as closely as possible, along with other desirable engineering / physics constraints



The PyPlasmaOpt Objective Function:

$$
\begin{aligned}
f(\mathbf{p}, \sigma, \iota_a, \mathbf{x}, \iota_s) = {} & \frac{1}{2}\left(\frac{L_c(\mathbf{p}) - L_{0,c}}{L_{0,c}}\right)^2 + \frac{1}{2}\left(\frac{L_a(\mathbf{p}) - L_{0,a}}{L_{0,a}}\right)^2 \\
& + \int_{\text{axis}} \|\boldsymbol{B}_{\text{coils}}(\mathbf{p}) - \boldsymbol{B}_{QS}(\mathbf{p})\|^2 \, dl + \int_{\text{axis}} \|\nabla \boldsymbol{B}_{\text{coils}}(\mathbf{p}) - \nabla \boldsymbol{B}_{QS}(\mathbf{p}, \sigma, \iota_a)\|^2 \, dl \\
& + \int_{\text{surface}} \left(\|\mathbf{B}_{\text{coils}}(\mathbf{x})\| - B_{\text{QS}}(\mathbf{p})\right)^2 \, da \\
& + \frac{1}{2}\left(\frac{\iota_a - \iota_{0,a}}{\iota_{0,a}}\right)^2 + \frac{1}{2}\left(\frac{\iota_s - \iota_{0,s}}{\iota_{0,s}}\right)^2
\end{aligned}
$$

where $\mathbf{g}_1(\mathbf{p}, \sigma, \iota_a) = \mathbf{0}$ and $\mathbf{g}_2(\mathbf{p}, \mathbf{x}, \iota_s) = \mathbf{0}.$

Courtesy A. Giuliani

Step 1: Boozer Coordinate Flux Surfaces

- Discretize PDEs using a spectral method
- Implement analytic jacobian calculation
- Solve using a newton method
- Ended up using a Levenberg-Marquardt implementation in Scipy
- 4 – 6 times speed up in residual / jacobian computation, only 2x speed increase in surface solve

$$\mathbf{B}_{\text{coils}} = \frac{B^2_{\text{coils}}}{G}\left(\frac{\partial \mathbf{x}}{\partial \theta} + \iota_s \frac{\partial \mathbf{x}}{\partial \varphi}\right)$$

$$\int_0^{2\pi}\int_0^{2\pi}\left\|\frac{\partial \mathbf{x}}{\partial \theta} \times \frac{\partial \mathbf{x}}{\partial \varphi}\right\| \, d\varphi \, d\theta = r$$

Step 2: Gradient of QS Surface Term in Obj Func.

- Use a forward sensitivity approach to calculate the gradient
- Currently implementing computation of sensitivity of surface residual wrt coil input parameters

$$\frac{\partial f}{\partial \mathbf{p}} = -\frac{\partial f}{\partial \mathbf{q}}\left[\frac{\partial \mathbf{g}}{\partial \mathbf{q}}^{-1} \boxed{\frac{\partial \mathbf{g}}{\partial \mathbf{p}}}\right] + \frac{\partial f}{\partial \mathbf{p}}$$

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